Analysis and discussion

1. Forward Euler Behavior:

The Forward Euler method is an explicit numerical method that approximates the solution using

And it’s conditionally stable, it requires a small step size “h” to maintain accuracy. As time Increases, the numerical solution diverges. This instability arises because the method does not handle stiff equations well, making it unstable for large step sizes.

1. Modified Euler Behavior:

The modified Euler method is an explicit numerical method as well, with a predictor-corrector approach that improves the Forward Euler method

Modified Euler is more stable than Forward Euler because it accounts for the slope at both the beginning and the estimated endpoint of the step. It provides a more accurate approximation since it reduces the errors per step.

1. Backward Euler Behavior:

The backward Euler method is an Implicit method where

For , this results it is unconditionally stable it does not produce oscillations, unlike explicit methods, solving Backward Euler requires an implicit solution.

1. Step Size Impact:

* Forward Euler: if “h” is too large, the method becomes unstable, leading to oscillations.
* Modified Euler: increasing “h” reduces accuracy but extends stability slightly compared to Forward Euler.
* Backward Euler: unconditionally stable for any “h”.
* Relation to Stability Region
* Forward Euler has strict stability conditions.
* Modified Euler improves stability slightly.
* Backward Euler is always stable, regardless of “h”.

1. Explicit VS Implicit methods:

* Explicit Methods:
* Computes directly from known values.
* less stable for stiff problems.
* Implicit Methods
* Require solving an equation for  *.*
* More stable for stiff ODEs.

1. Stability Condition for Forward Euler:

For a general linear ODE: Forward Euler is stable if :

Derivation of stability condition:

Using the forward Euler formula for

Stability, we need:

for , we get:

* If , forward Euler becomes unstable.
* If , the solution remain somewhat stable but not accurate.